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SUPERSYMMETRIC GRAND UNIFIED THEORIES AND YUKAWA UNIFICATION

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INTRODUCTION

In this paper, I intend to motivate supersymmetric grand unified theories (SUSY GUTs), briefly explain an extension of the standard model based on them and present a calculation performed using certain properties of some SUSY GUTs to constrain the available parameter space.

Why GUTs?

Much work has been done on the running of the gauge couplings in the standard model, as prescribed by the renormalisation group. Amazingly, when the couplings α_1, α_2 and α_3 were run up to fantastically high energies $\sim 10^{14}$ GeV, they seemed to be converging[1] to one value. This is a feature naturally explained by many GUTs such as SU(5)[2,3] and reflects the fact that the strong, weak and electromagnetic forces seen today are different parts of the same grand unified force. It was realised that GUTs could also provide relations between the masses of the observed fermions, the structure and hierarchy of which are as yet unexplained. Despite these attractive features, several problems arose which detracted from the idea.

Unfortunately, the three couplings do not quite converge by $\sim 7\sigma$, and many GUTs, notably SU(5), predict proton decay much faster than the lower experimental bounds. Also incredible fine tuning is required for the so-called 'hierarchy problem'. This stems from the fact that M_W changes through radiative corrections of order the new physics scale (Fig.1) , say the Planck mass $\sim 10^{19}$ GeV, if there is no new physics at smaller energies. M_W is therefore unstable to the corrections and vast cancellations in the couplings are required to motivate the correct phenomenology.

Figure 1: One loop corrections to m_h^2. The first diagram gives a $\sim M_{Pl}^2$ contribution.

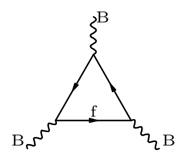

Figure 2: One loop triple hypercharge boson anomaly.

Why SUSY?

Supersymmetry is an extra symmetry relating fermions and bosons, and so provides some explanation of how particles of differing spin should be related to one another. In an unbroken supersymmetric theory, each fermion has a degenerate bosonic partner. Of course, these so called superpartners are not observed, so that if supersymmetry was ever the correct theory, it must have been broken. However, with the introduction of superpartners at some rough energy scale M_{SUSY}, the renormalisation group running of the gauge couplings changes. The coupling constants are now seen to meet at a scale $\sim 10^{16}$ GeV, as reflected by the correct $\sin^2\theta_w$ prediction[4]. M_W becomes stabilised because supersymmetry induces cancellations between the bosonic and fermionic loop corrections to the mass. The quadratic divergences induced by the loop corrections now add to zero and one is left with merely logarithmic divergences.

THE MINIMAL SUPERSYMMETRIC STANDARD MODEL (MSSM)

The MSSM is a minimal extension of the standard model into supersymmetry. In the model, every particle of the standard model has a superpartner associated with it that transforms identically under the standard model gauge group but have spin different by $\frac{1}{2}$. So for example, each quark has a scalar "squark" superpartner, the gluons have "gluinos" etc. At first sight however, the model has a $\mathrm{U}(1)_Y^3$ gauge anomaly. This originates from the diagram with three B gauge bosons connected to an internal loop through which any fermions may run (cf Fig.2) and the counter term to it would destroy gauge invariance. The diagram is proportional to $\sum_i (Y_i/2)^3$ where i runs over all active fermions. Through the hypercharge assignments, this cancels in the standard model but in the MSSM the superpartner of the Higgs called the Higgsino with $Y = 1$ may run around the loop. To cancel this effect, a second Higgs H_2 must be introduced which transforms in the same way to H_1 except for having $Y = -1$.

The new Higgs must also develop a vev v_2 to give masses to up quarks and the two vevs are related by

$$\tan\beta = \frac{v_2}{v_1} \tag{1}$$

where $v_1^2 + v_2^2 = v^2$ and $v = 246$ GeV, the measured vev of the standard model.

In chiral superfield form, the superpotential looks like

$$W_{MSSM} = UQH_2u^c + DQH_1d^c + ELH_1e^c + \mu H_1H_2 \tag{2}$$

where U, D and E are the up, down and charged lepton Yukawa matrices respectively and all gauge and family indices have been suppressed.

One possible problem with this superpotential is the dimensionful parameter μ. μ needs to be $\sim M_Z$ to give the right electroweak symmetry breaking behaviour whereas one would expect it to be of order of the new physics scale M_{GUT}. One solution to this problem is described in the Next to Minimal Supersymmetric Standard Model (NMSSM).

The NMSSM

The μ term in Eq.2 is replaced by λNH_1H_2 where N is a gauge singlet and therefore doesn't affect the coupling constant unification. In certain supergravity models, N develops a vev naturally of order M_Z and so the μ term is generated without having to put μ in "by hand." The superpotential now has a discrete Peccei-Quinn symmetry which leads to phenomenologically unacceptable low energy axions and so a term $-\frac{k}{3}N^3$ is added which breaks it. [1]

GUTS WITH YUKAWA UNIFICATION

GUTs can quite naturally provide Yukawa unification relations between the quarks and/or leptons. For example in SU(5), the right handed down quarks and conjugated lepton doublet lie in a $\underline{5}$ representation. When a mass term $\sim 5^i5_i$ is formed, the Yukawa relation

$$\lambda_b(M_{GUT}) = \lambda_\tau(M_{GUT}) \tag{3}$$

applies. Also in SO(10), the whole of one family and a right handed neutrino is contained in one $\underline{16}$ representation, leading to triple Yukawa unification, where the top, bottom and charged lepton Yukawa couplings are equal at the GUT scale.

These relations can be used to constrain the parameter space of m_t and $\tan\beta$, which has been done for the MSSM[5]. Our idea was to repeat this calculation for the NMSSM, to see how much the viable parameter space changes in the model.

THE CALCULATION

The basic idea is to choose some $\tan\beta$ and m_t and run λ_b and λ_τ up to $M_{GUT} \sim 10^{16}$ GeV. Then, to some arbitrary accuracy, one can determine whether the GUT relation Eq.3 holds. If it does, then SU(5) and the other Yukawa unifying extensions of the standard model are possible on this point in parameter space. The procedure is iterated over all reasonable values of $\tan\beta$ and m_t. The calculation is presented in more detail in Ref.6.

[1] λ and k are merely coupling constants

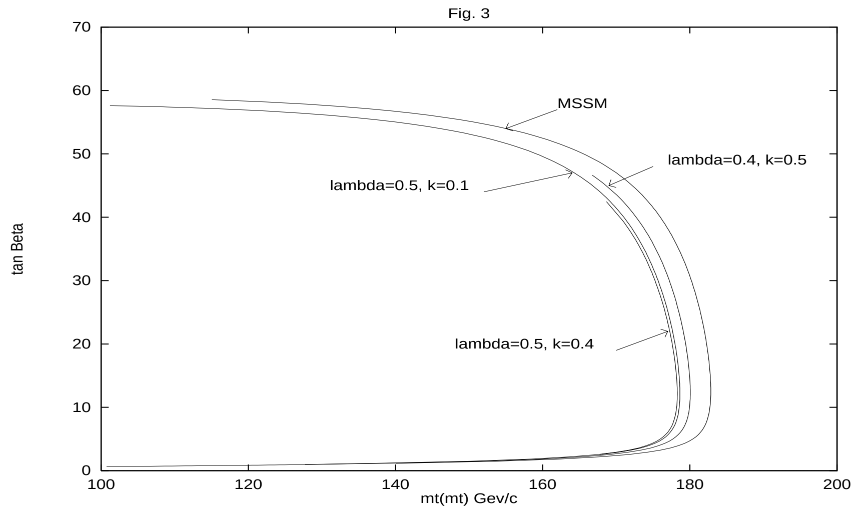

Figure 3: Viable Range of Parameter Space For $\alpha_S(M_Z) = 0.11$, $m_b = 4.25$ GeV. λ and k values are quoted at m_t.

Starting Point M_Z.

We use the definitions of the gauge couplings at M_Z: $\alpha_1^{-1}(M_Z) = 58.89$, $\alpha_2^{-1}(M_Z) = 29.75$ and $\alpha_3^{-1}(M_Z) = 0.11 \pm 0.01$. The first two gauge couplings are determined accurately enough for our purposes whereas the third needs to be used as a parameter, on account of its large uncertainty.

In order to convert masses of quarks to Yukawa couplings, we simply need to read them off the potential Eq.2 at some energy scale (taken here to be m_t):

$$\lambda_t\left(m_t\right) = \frac{\sqrt{2}m_t\left(m_t\right)}{v\sin\beta} \tag{4}$$

$$\lambda_b\left(m_t\right) = \frac{\sqrt{2}m_b\left(m_b\right)}{\eta_b v\cos\beta} \tag{5}$$

$$\lambda_\tau\left(m_t\right) = \frac{\sqrt{2}m_\tau\left(m_\tau\right)}{\eta_\tau v\cos\beta}. \tag{6}$$

where

$$\eta_f = \frac{m_f\left(m_f\right)}{m_f\left(m_t\right)}. \tag{7}$$

Note that whereas the m_t referred to here is always the running one, it can be related to the physical mass by[5]

$$m_t^{phys} = m_t\left(m_t\right)\left[1 + \frac{4}{3\pi}\alpha_3\left(m_t\right) + O\left(\alpha_3^2\right)\right]. \tag{8}$$

To determine η_b and η_τ, the masses are run up from the on shell mass to m_t using effective 3 loop QCD $\otimes$ 1 loop QED [7,8,9,10]. Note that these factors will depend of $m_b = 4.25 \pm 0.15$ GeV and $\alpha_3(M_Z)$. m_t is assumed to be the rough energy scale when the whole supersymmetric spectrum kicks in. While being unrealistic, trials with $M_{SUSY} = 1$ TeV show only a few percent deviation from the predictions with $M_{SUSY} = m_t$. So, having determined the gauge and relevant Yukawa couplings at m_t,

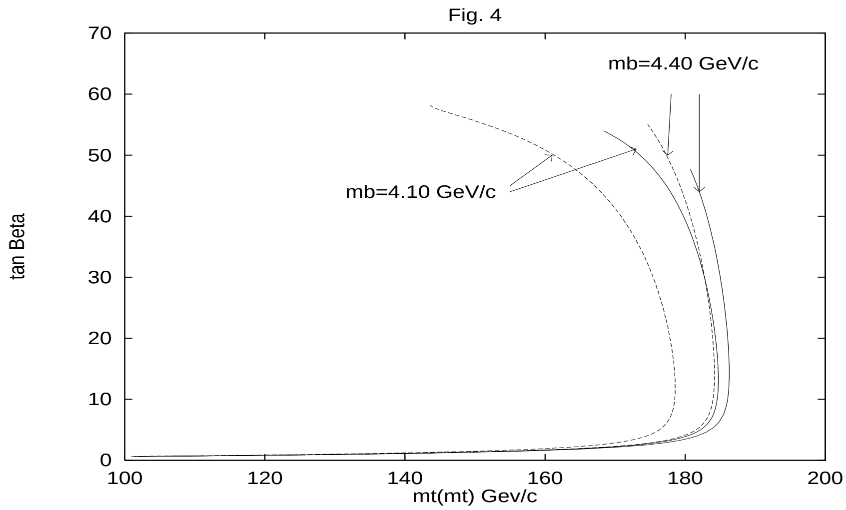

Figure 4: Viable Range of Parameter Space For $\alpha_S(M_Z) = 0.12$ and experimental bounds of $m_b = 4.1$–4.4 GeV. The left most lines are for $\lambda_b = 0.9\lambda_\tau$.

we need RG equations to run them up to M_{GUT} in the NMSSM. To derive these, we used results from a general superpotential[11] to obtain

$$\begin{aligned}
16\pi^2 \frac{\partial \lambda_t}{\partial t} &= \lambda_t \left[6\lambda_t^2 + \lambda_b^2 + \lambda^2 - \left(\frac{13}{15} g_1^2 + 3g_2^2 + \frac{16}{3} g_3^2 \right) \right] \\
16\pi^2 \frac{\partial \lambda_b}{\partial t} &= \lambda_b \left[6\lambda_b^2 + \lambda_\tau^2 + \lambda_t^2 + \lambda^2 - \left(\frac{7}{15} g_1^2 + 3g_2^2 + \frac{16}{3} g_3^2 \right) \right] \\
16\pi^2 \frac{\partial \lambda_\tau}{\partial t} &= \lambda_\tau \left[\lambda_\tau^2 + 3\lambda_b^2 + \lambda^2 - \left(\frac{9}{5} g_1^2 + 3g_2^2 \right) \right] \\
16\pi^2 \frac{\partial \lambda}{\partial t} &= \lambda \left[4\lambda^2 + 2k^2 + 3\lambda_\tau^2 + 3\lambda_b^2 + 3\lambda_t^2 - \left(\frac{3}{5} g_1^2 + 3g_2^2 \right) \right] \\
16\pi^2 \frac{\partial k}{\partial t} &= 6k \left[\lambda^2 + k^2 \right] \qquad (9)
\end{aligned}$$

in the limit that the lighter two families have negligible contributions (a very good approximation).

The Yukawa couplings can now be run from m_t to 10^{16} GeV using numerical techniques. The parameters λ and k particular to the NMSSM are unconstrained at m_t so they are merely varied for different curves.

Our results are displayed in Fig. 3 as contours in the $\tan\beta - m_t$ plane consistent with Eq.3. We take $\alpha_3(M_Z) = 0.11$, $m_b = 4.25 GeV$ and the NMSSM parameters $\lambda(m_t)$ and $k(m_t)$ as indicated. The MSSM contour is shown for comparison and is indistinguishable from the NMSSM contour with $\lambda(m_t) = 0.1$ and $k(m_t) = 0.5$. In fact our plot for the MSSM based on 1-loop RG equations is very similar to the 2-loop result in ref.5. The deviation of the NMSSM contours from the MSSM contour depends most sensitively on $\lambda(m_t)$ rather than $k(m_t)$. Two of the contours are shortened due to either λ or k blowing up at the GUT scale. For $\lambda(m_t) = 0.5$, $k(m_t) = 0.5$, no points in the $m_t - \tan\beta$ plane are consistent with Eq.3 Yukawa unification, while for $\lambda(m_t) = 0.1$, $k(m_t) = 0.1 - 0.5$ the contours are virtually indistinguishable from the MSSM contour. In general we find that for any of the current experimental limits on α_3 and m_b, the maximum value of $\lambda(m_t)$ or $k(m_t)$ is ~ 0.7 for a perturbative solution to Eq.3.

Fig.4 shows the effects of particle thresholds, which can modify Eq.3 to $\lambda_b = 0.9\lambda_\tau$. Our treatment does not treat supersymmetric or heavy thresholds exactly and so some

sort of corrections like those shown are expected. The curves are at $\alpha_S(M_Z) = 0.12$ and $m_b = 4.1$–4.4 GeV to illustrate that uncertainties in these quantities make a large difference to the parameter space. These uncertainties are much bigger than those associated with the NMSSM, and so the MSSM and NMSSM would be practically indistinguishable given the parameters m_t and $\tan\beta$.

Other Yukawa Parameters

The next useful step is to notice that Eqs.9 are all of the form

$$16\pi^2 \frac{\partial \lambda_a}{\partial t} = \lambda_a \left[\sum_i M_i^a \lambda_i^2 - \sum_{j=1}^{3} c_j^a g_j^2 \right], \tag{10}$$

where M_i^a and c_j^a are constants supplied by the relevant RG equation. When the β function

$$\frac{dg_i}{dt} = \frac{b_i g_i^3}{16\pi^2} \tag{11}$$

is inserted, and the RG equations are reparameterised in terms of the flow and not the trajectory of the solutions, we obtain

$$\frac{\lambda_a(M_{SUSY})}{\lambda_a(M_{GUT})} = \xi^a \exp(-\sum_i M_i^a I_i), \tag{12}$$

where

$$\xi^a = \prod_{i=1}^{3} \left(\frac{\alpha(M_{GUT})}{\alpha_i(M_{SUSY})} \right)^{\frac{c_i^a}{2b_i}} \tag{13}$$

contains all the information about the gauge couplings and

$$I_i = \frac{1}{16\pi^2} \int_{\ln(M_{SUSY})}^{\ln(M_{GUT})} \lambda_i^2 \mathrm{d}t \tag{14}$$

concerns the Yukawa couplings.

With this formulation, the running of the physically relevant Yukawa eigenvalues and mixing angles can be expressed in simple terms as shown below,

$$\begin{aligned}
\left(\frac{\lambda_{u,c}}{\lambda_t} \right)_{M_{SUSY}} &= \left(\frac{\lambda_{u,c}}{\lambda_t} \right)_{M_{GUT}} e^{3I_t + I_b} \\
\left(\frac{\lambda_{d,s}}{\lambda_b} \right)_{M_{SUSY}} &= \left(\frac{\lambda_{d,s}}{\lambda_b} \right)_{M_{GUT}} e^{3I_b + I_t} \\
\left(\frac{\lambda_{e,\mu}}{\lambda_\tau} \right)_{M_{SUSY}} &= \left(\frac{\lambda_{e,\mu}}{\lambda_\tau} \right)_{M_{GUT}} e^{3I_\tau} \\
\frac{| V_{cb} |_{M_{GUT}}}{| V_{cb} |_{M_{SUSY}}} &= e^{I_b + I_t},
\end{aligned} \tag{15}$$

with identical scaling behaviour to V_{cb} of V_{ub}, V_{ts}, V_{td}. To a consistent level of approximation V_{us}, V_{ud}, V_{cs}, V_{cd}, V_{tb}, λ_u/λ_c, λ_d/λ_s and λ_e/λ_μ are RG invariant. The CP violating quantity J scales as V_{cb}^2. Eqs. 15, 14 also apply to the NMSSM since the extra λ and k parameters cancel out of the RG equations in a similar way to the gauge contributions as can easily be seen from Eq.9. The only difference to these physically relevant quantities is therefore contained in I_τ, I_b and I_t.

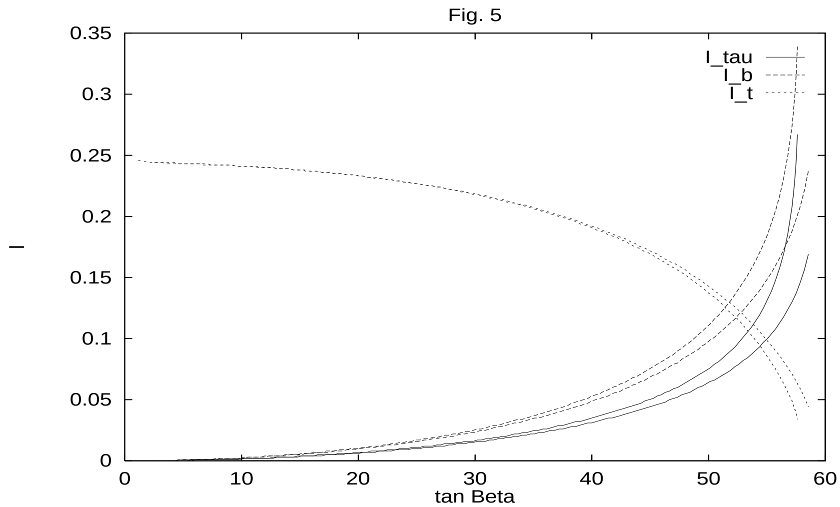

Figure 5: I_i integrals for $\alpha_S(M_Z) = 0.11$, $m_b = 4.25$ GeV.

These I_i integrals are shown in Fig.5 and the NMSSM results are the upper lines of each pair, and it is clear that the deviation between the two models is small again.

We emphasise that the results of the I_i integrals shown in Fig.5 play a key role in determining the entire fermion mass spectrum via the scaling relations of Eq.15. The small deviation between the NMSSM and the MSSM results compared to the experimental uncertainties means that the recent GUT scale texture analyses of the quark mass matrices which were performed for the MSSM are equally applicable to the NMSSM. For example, the recent Ramond, Roberts and Ross (RRR)[12] texture analysis is also based upon Eq.3 and assumes a Georgi-Jarlskog (GJ)[13,14] ansatze for the charged lepton Yukawa matrices, although their results in the quark sector are insensitive to the lepton sector. It is clear that all the RRR results are immediately applicable to the NMSSM since the only difference between the two models enters through the scaling integrals I_i whose deviation we have shown to be negligible compared to the experimental errors.

CONCLUSIONS

We have discussed the unification of the bottom quark and tau lepton Yukawa couplings within the framework of the NMSSM. By comparing the allowed regions of the m_t-$\tan\beta$ plane to those in the MSSM we find that over much of the parameter space the deviation between the predictions of the two models which is controlled by the parameter λ is small, and always much less than the effect of current theoretical and experimental uncertainties in the bottom quark mass and the strong coupling constant. We have also discussed the scaling of the light fermion masses and mixing angles, and shown that to within current uncertainties, the results of recent quark texture analyses[12] performed for the minimal model also apply to the next-to-minimal model. There are however two distinguishing features of the NMSSM. Firstly, the scaling of the charged lepton masses will be somewhat different, depending on λ and k. Although this will not affect the quark texture analysis of RRR, it may affect the success of the GJ ansatze[13,14] for example. Secondly, the larger $\tan\beta$ regions may not be accessible in the NMSSM for large values of λ and k, so that full Yukawa unification may not be possible in this case.

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